INVENTORY CONTROL UNDER DEMAND SUBSTITUTION : SOME NEW ADDITIONS

By

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CERTIFICATE

This is to certify that the present work on INVENTORY CONTROL UNDER DEMAND SUBSTITUTION: SOME NEW ADDITIONS, by A. Venkatasubramanian, has been carried out under my supervision and has not been submitted elsewhere for the award of a degree.

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(A. Venkatasubramanian)

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ABSTRACT

The present work on substitution deals with various models where the interaction of demand due to shortages and/or introduction of new items is taken into account in two-item/multi-item inventory system.

Deterministic and stochastic models have been developed for various situations. In deterministic case, two-way complete substitution of two items of decaying nature is considered and a solution methodology is suggested. A simulation model to solve the problem of single item one-way substitution by maintaining end-item substitute and second level item inventories as safety stocks along with emergency ordering for end-item substitute to meet a particular service level is developed. A simulation model for two-way substitution of N-items with periodic review policy is studied and an iterative method to find the decision variables of the model is described.

Solution methodology has been suggested in all cases. Numerical examples are demonstrated to show the steps involved.

CHAPTER I

INTRODUCTION

1.1 SUBSTITUTION - AN OVER VIEW:

Organisations try to boost sales by improving quality of the product at a lower price and by maintaining an uninterrupted supply of their items. In recent years, the whispering problems of buyers being the non-availability of some durable products and their sub-components in time. Eventhough industries have been using mathematical models, market survey methods to predict the demand for their items in beforehand, there is no single cost-effective model to give exact demand for a product. In an unstable world where industries are appearing and disappearing in a short period of time, the demand for a product of particular brand goes naturally to some other manufacturer, resulting in unpredictable demand fluctuations which are reported to be as high as 30 percent of the calculated demand in rare cases.

In such a market environment fraught with uncertainties of demand and supply, substituting an item for another item with similar characteristics with inferior or superior qualities, becomes inevitable to maintain goodwill of the customer and to build the rapport. The above said environment typically prevails in Indian Market.

Eventhough it is in this context that the problem of substitution is important, industries want to be cautious about the substitution policy. In an effort to improve goodwill of the customer and reputation, organisations do not want to take the risk of over substitution which has got its negative aspects also. By putting some policy restrictions and by maintaining an uninterrupted supply of the items in market, in the aforesaid situations, they avert problems.

1.2 ORGANISATION OF THESIS:

The present study deals with single item as well as multi-item inventory system under substitutable demand situations.

In Chapter II, for complete substitution of two items for each other where items are of decaying in nature, a mathematical model is developed, and solution methodology is given. A practical situation of a single item inventory system with emergency set-up cost, reworking cost is dealt and a simulation model for finding optimal inventory policy to meet a required service level is given in Chapter III. The Problem of N-item substitution with (Q,r,T) model is described in Chapter IV and solution methodology to find the values of decision variables the periodic review model with a simulation approach is explained in detail.

For all the models dealt in this present work, numerical examples are given and results are analysed.

CHAPTER II

TWO ITEM SUBSTITUTION WITH DECAY

2.1 PROBLEM STATEMENT:

Many organic compounds, medicinal items and fashion goods deteriorate as time passes. This is reported to be due to the presence of some chemicals without which the compounds cannot be prepared. Sometimes the deterioration is accelerated by the amount stored and/or by other catalysts present in the items.

We consider an inventory system of a retailer of such items which are of decaying in nature. Among them, let us assume that few items have similar characteristics in terms of their utilities, purchase costs etc., and can be put in a group. We now concentrate on a group of two items which can substitute each other. It is assumed that here in the situation of stockout of one item, its demands can be completely satisfied by the other. And there will be atleast one item in the stock at any time to avoid shortages.

In this chapter, a simple inventory system with constant demand rate, infinite rate of production and variable rate of deterioration is considered where items are of substitutable in nature. A special form of two parameter Weibull distribution is used to represent deterioration rate, which is reported to be realistic by Deb and Chaudhuri[2]. The demand of one item is completely fulfilled by the other item and vice versa. In other words, at any time the system has the stock of only one item.

A model is developed with the assumptions mentioned above and a numerical illustration is presented.

2.2 LITERATURE REVIEW:

Inventory models for items deteriorating with time have engaged attention of researchers in recent years. Various attempts have been made to account for decay of items in inventory systems. An EOQ model for an inventory with a constant rate of deterioration was developed by Ghare and Schroder[3]. An order-level inventory model for items with uniform rate of deterioration was introduced by Shah and Jaiswal [6]. In the recent publication, an EOQ model for items with finite rate of production and variable rate of deterioration has been reported by Deb and Chaudhuri [2]. Eventhough, many papers have been published, based on deterioration, literature is lacking in substitution of items which are deteriorating at a variable rate.

2.3 ASSUMPTIONS:

The following assumptions are made in designing the model.

1) Demand rate of each item is deterministic and static.
2) The replenishment is instantaneous.

3) At the time of procurement of item 1, stock levels of items 1 and 2 are zero. Similarly at the time of procurement of item 2, stock levels of items 1 and 2 are zero.

4) No shortage is allowed. As items are mutually interchangeable complete substitution of one item by other leads to no shortage situation.

5) Planning horizon is infinite.

6) Unit variable cost of any items does not depend upon the replenishment quantities. In other words, no quantity discount is allowed.

7) The substitution cost is directly proportional to the number of units to be substituted.

8) Rate of deterioration follows a special form of Weibull distribution to make the problem mathematically tractable.

2.4 NATURE OF DETERIORATION:

Two-parameter Weibull distribution is taken to represent deterioration rate.

Weibull distribution function is given as,

$$\theta(t) \;=\; \lambda\,\beta(\lambda t)^{\beta-1}\; e^{-(\lambda t)^{\beta}} \qquad t \geq 0$$

where λ and β are positive constants.

To make the problem mathematically solvable, a special form is considered by putting $\beta = 2$, we get,

$$\theta(t) \;=\; 2\lambda(\lambda t)^{1}\;\; e^{-(\lambda t)^{2}}$$

$$\;=\; 2\lambda^{2} t\;(1 - \frac{\lambda^{2} t^{2}}{1} + \frac{\lambda^{4} t^{4}}{2} + \ldots\;)$$

By dropping higher powers of t (as $(\lambda t)^n \rightarrow 0$ as $n \rightarrow \infty$ and $\lambda \rightarrow 0$), we get,

$$\theta(t) = \alpha t \qquad t \geq 0$$

where $2\lambda^2 = \alpha$.

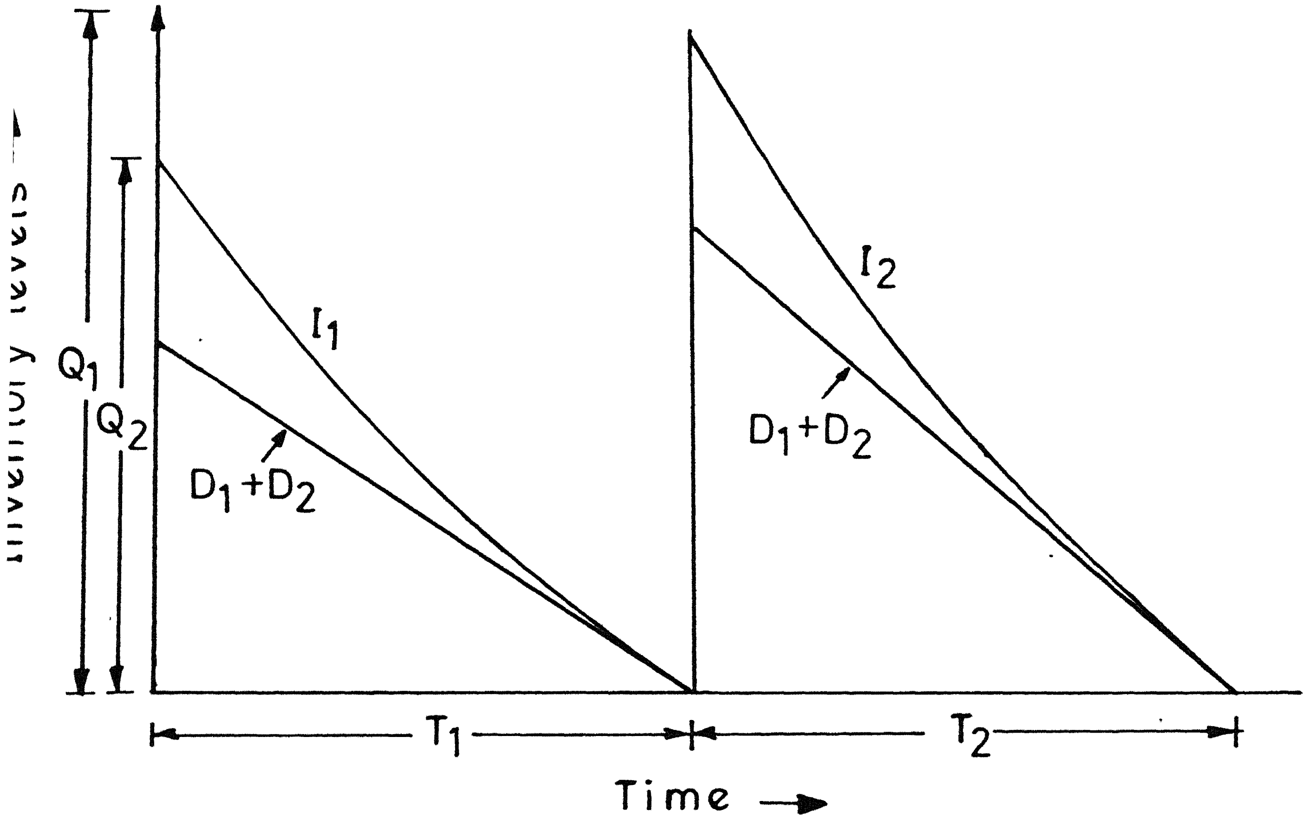
Here $0 < \alpha < < 1$ for practical purposes [2].

Thus deterioration rate is linear with respect to time.

2.5 NOMENCLATURE:

Parameters

D_1	Demand of item 1
D_2	Demand of item 2.
$\theta_1(t) = \alpha_1 t$	Deterioration rate of item 1 at time t $t \geq 0$.
$\theta_2(t) = \alpha_2 t$	Deterioration rate of item 2 at time t $t \geq 0$.
h_1	Holding cost of item 1, in Rs./unit/unit time.
h_2	Holding cost of item 2 in Rs./unit/unit time.
C_1	Unit cost of item 1.
C_2	Unit cost of item 2.
A_1	Fixed ordering cost of item 1 in Rs./order.
A_2	Fixed ordering cost of item 2 in Rs./order.
V_1	Substitution cost of item 1 when it is substituted by item 2 in Rs./unit.
V_2	Substitution cost of item 2 when it is substituted by item 1 in Rs./unit.
F_1	Loss per unit item decay of item 1 in Rs./unit.
F_2	Loss per unit item decay of item 2 in Rs./unit.



I_1-Instantaneous level of inventory of item 1

I_2-Instantaneous level of inventory of item 2

Fig. 2·1 Two way substitution of two items
(For items of decaying nature)

Decision Variables:

Q_1	The replenishment quantity of item 1 per cycle in units.
Q_2	The replenishment quantity of item 2 per cycle in units.

Objective Function:

TC	Total annual cost of both items consisting of inventory holding cost, procurement cost (fixed and variable) and substitution and loss of decay costs.

2.6 FORMULATION:

Fig. 2.1 shows the behaviour of on-hand inventory with time. At the time of procurement of item 1 stock level of both items are zero. Inventory level of item 1 is raised to Q_1 and demand for both items are met with item 1.

After stockout of item 1, item 2 is procured. This will meet the demand of items 1 and 2 for the period of duration T_2.

This cycle of length $T_1 + T_2$ is repeated subsequently.

Referring to Fig. 2.1, the instantaneous level of inventory is derived as follows. Let $Q_1(t)$ be the inventory level at time t ($0 \leq t < T_1$). During the time interval $(t, t+\Delta t)$, the inventory level will deplete by an amount $\theta_1(t).Q_1(t) \cdot \Delta t$ due to decay and by an amount $(D_1+D_2)\ \Delta t$ due to demand. Thus,

$$Q_1(t+\Delta t) = Q_1(t) - \theta_1(t).Q_1(t).\Delta t - (D_1+D_2)\ \Delta t$$

Transposing and dividing by Δt, we obtain,

$$\frac{Q_1(t+\Delta t) - Q_1(t)}{\Delta t} = - \theta_1(t)\; Q_1(t) - (D_1+D_2)$$

Taking limits as $\Delta t \rightarrow 0$ yields,

$$\frac{dQ_1(t)}{dt} = - \theta_1(t)\; Q_1(t) - (D_1+D_2) \qquad 0 \leq t < T_1$$

By rearranging we get,

$$\frac{dQ_1(t)}{dt} + \theta_1(t)\; Q_1(t) = - (D_1+D_2) \qquad 0 \leq t < T_1 \qquad (2.1)$$

As the decay of item 2 starts only after its procurement the instantaneous level of inventory for item 2 is determined following the same approach as for item 1 and is given below:

$$\frac{dQ_2(t)}{dt} + \theta_2(t)\; Q_2(t) = - (D_1+D_2) \qquad 0 \leq t < T_2 \qquad (2.2)$$

Total time per cycle, $T = T_1 + T_2$.

By substituting $\theta_1(t) = \alpha_1 t$ and $\theta_2(t) = \alpha_2 t$ in equns.(2.1) and (2.2) we get,

$$\frac{dQ_1(t)}{dt} + \alpha_1 t\; Q_1(t) = - (D_1+D_2) \qquad (2.3)$$

$$\frac{dQ_2(t)}{dt} + \alpha_2 t\; Q_2(t) = - (D_1+D_2) \qquad (2.4)$$

Solving the differential equation (2.3), we get,

$$Q_1(t) = -(D_1+D_2)\; \{e^{-\frac{\alpha_1}{2} t^2}\}\; [t + \frac{\alpha_1}{6} t^3] + K_1\, e^{-\frac{\alpha_1}{2} t^2} \qquad (2.$$

where K_1 is constant of integration.

At $t = 0$, $Q_1(t) = Q_1$. With this initial condition, $K_1 = Q_1$. Substituting $K_1 = Q_1$ in eqn. (2.5), and substituting the expansion for $\exp\left(-\frac{\alpha_1}{2} t^2\right)$, we get,

$$Q_1(t) = -(D_1+D_2)\left(t - \frac{\alpha_1}{3} t^3\right) + Q_1\left(1 - \frac{\alpha_1}{2} t^2\right)$$

At $t = T$, $Q_1(t) = 0$. Substituting this in eqn. (2.5), we get,

$$\frac{\alpha_1}{3} (D_1+D_2)\, T_1^3 - \frac{\alpha_1}{2} Q_1 T_1{}^3 - (D_1+D_2)\, T_1 + Q_1 = 0$$

After simplification, we get,

$$T_1{}^3 + pT_1{}^2 + qT_1 + r = 0 \tag{2.6}$$

where,

$$p = -3Q_1/(2(D_1+D_2))$$
$$q = -3/\alpha_1$$
$$r = 3Q_1/(\alpha_1(D_1+D_2))$$

By substituting $T_1 = x_1 - \frac{p}{3}$ in eqn. (2.6), we get,

$$x_1{}^3 + ax_1 + b = 0 \tag{2.7}$$

where,

$$a = \frac{1}{3}(3q-p^2)$$
$$= -3\left(\frac{\alpha_1 Q_1{}^2 + 4(D_1+D_2)^2}{4\alpha_1\,(D_1+D_2)^2}\right)$$

$$b = \frac{1}{27}(2p^3 - 9pq + 27r)$$

$$= \frac{6Q_1(D_1+D_2)^2 - \alpha_1 Q_1{}^3}{4\alpha_1\,(D_1+D_2)^3}$$

Solution for $x_1^3 + ax_1 + b = 0$ is

$$x_1 = A_1+B_1, \ -\frac{A_1+B_1}{2} + \frac{A_1-B_1}{2}(\sqrt{-3}), \ -\frac{A_1+B_1}{2} - \frac{A_1-B_1}{2}(\sqrt{-3}) \tag{2.8}$$

where,

$$A_1 = \left(\frac{b}{2} + \sqrt{\frac{b^2}{4} + \frac{a^3}{27}} \right)^{1/3}$$

$$B_1 = \left(\frac{b}{2} - \sqrt{\frac{b^2}{4} + \frac{a^3}{27}} \right)^{1/3}$$

After simplification, we get,

$$A_1 = [K_1Q_1^3+K_2Q_1 + \sqrt{K_3Q_1^6+K_4Q_1^4+K_5Q_1^2+K_6}\ ]^{1/3} \tag{2.9}$$

$$B_1 = [K_1Q_1^3+K_2Q_1 - \sqrt{K_3Q_1^6+K_4Q_1^4+K_5Q_1^2+K_6}\ ]^{1/3} \tag{2.10}$$

where,

$K_1 = -1/8(D_1+D_2)$
$K_2 = 3/4(D_1+D_2)$
$K_3 = 0.0$
$K_4 = -3/(8(D_1+D_2)^4\ \alpha_1)$
$K_5 = -3/(16\alpha_1^2(D_1+D_2)^2)$
$K_6 = -1/\alpha_1^3$

For making the analysis easy, rewriting the roots of the equation (2.7) in different form, we get,

$$x_1 = y_1(A_1+B_1) - z_1(A_1-B_1) \tag{2.11}$$

where, (y_1, z_1) can take

$(1.0,0.0)$ or $(-0.5,0.866\,\overset{i}{\wedge})$ or $(-0.5,-0.866\,\overset{i}{\wedge})$

Let $A_{11} = K_1Q_1^{\,3} + K_2Q_1$ (2.12)

$B_{11} = (-1(K_4Q_1^{\,4} + K_5Q_1^{\,2} + K_6))^{1/2}$ (2.13)

As constants K_1 to K_6 are negative and values within square root terms of A_1 and B_1 are negative, We will be getting real roots after separating real and imaginary parts.

Rewriting values of A_1 and B_1 in eqn. (2.9) and (2.10) we get,

$$A_1 = (A_{11}^2 + B_{11}^2)^{1/6} \left(\text{Cos}\left(\frac{C}{3}\right) + i\ \text{Sin}\left(\frac{C}{3}\right)\right)$$

$$B_1 = (A_{11}^2 + B_{11}^2)^{1/6} \left(\text{Cos}\left(\frac{C}{3}\right) - i\ \text{Sin}\left(\frac{C}{3}\right)\right)$$

where, $C = \tan^{-1} (B_{11}/A_{11})$

Finding values for A_1+B_1 and A_1-B_1, we get,

$$A_1+B_1 = 2(A_{11}^2+B_{11}^2)^{1/6}\ \text{Cos}\ (C/3) \qquad (2.14)$$

$$A_1-B_1 = 2(A_{11}^2+B_{11}^2)^{1/6}\ \text{Sin}\ (C/3) \times i \qquad (2.15)$$

Eqn. (2.11) will become,

$$x_1 = 2y_1\ (A_{11}^2+B_{11}^2)^{1/6}\ \text{Cos}\ (C/3) - 2z_1 i (A_{11}^2+B_{11}^2)^{1/6}\ \text{Sin}\ (C/3) \qquad (2.16)$$

By putting $T_1 = x_1 - \frac{p}{3}$, we can find 3 values of T_1.

For item 2, solution methodology for solving the eqn.(2.2) is same as for eqn. (2.1).

Finally we get,

$$x_2 = 2y_2 \left(A_{21}^2 + B_{21}^2\right)^{1/6} \mathrm{Cos}\left(\frac{C_1}{3}\right)$$

$$- 2z_2 i \left(A_{21}^2 + B_{21}^2\right)^{1/6} \mathrm{Sin}\left(\frac{C_1}{3}\right) \qquad (2.17)$$

where,

$$A_{21} = R_1 Q_2^{\,3} + R_2 Q_2$$

$$B_{21} = \left(-\left(R_4 Q_2^{\,4} + R_5 Q_2^{\,2} + R_6\right)\right)^{1/2}$$

$$C_1 = \tan^{-1} \frac{B_{21}}{A_{21}}$$

$$R_1 = -1/8(D_1+D_2)$$

$$R_2 = 3/4(D_1+D_2)$$

$$R_3 = 0.0$$

$$R_4 = -3/8(D_1+D_2)^4 \alpha_2$$

$$R_5 = -3/(16(D_1+D_2)^2 \alpha_2^2)$$

$$R_6 = -1/\alpha_2^{\,3}$$

and (y_2, z_2) can take values $(1.0, 0.0)$ or $(-0.5, 0.866i)$ or $(-0.5, -0.866i)$.

By substituting $T_2 = x_2 - \dfrac{Q_2}{2(D_1+D_2)}$

we get 3 values for T_2.

Formulation of Total Cost:

(a) Procurement Cost:

For item 1 $= A_1 + C_1Q_1$

For item 2 $= A_2 + C_2Q_2$

(b) Inventory Holding Cost:

For item 1 $= \int_0^{T_1} h_1Q_1(t)\,dt$

$$= \int_0^{T_1} h_1[Q_1(1- \frac{\alpha_1}{2} t^2) - (D_1+D_2)(t- \frac{\alpha_1}{3} t^3)]\,dt$$

$$= h_1\,[Q_1(t - \frac{\alpha_1}{6} t^3) - (D_1+D_2)(\frac{t^2}{2} - \frac{\alpha_1}{12} t^4)]_0^{T_1}$$

$$= h_1\,[Q_1(T_1 - \frac{\alpha_1T_1^3}{6}) - (D_1+D_2)(\frac{T_1^2}{2} - \frac{\alpha_1T_1^4}{12})]$$

For item 2, inventory holding cost

$$= h_2[Q_2\,(T_2 - \frac{\alpha_2T_2^3}{6}) - (D_1+D_2)(\frac{T_2^2}{2} - \frac{\alpha_2T_2^4}{12})]$$

(c) Substitution Cost:

For item 2 $= V_2\,D_2\,T_1$

For item 1 $= V_1\,D_1\,T_2$

(d) Loss due to decay of item 1

$$= \int_0^{T_1} F_1\alpha_1\,[Q_1(1- \frac{\alpha_1t^2}{2}) - (D_1+D_2)(t- \frac{\alpha_1t^3}{3})]\,dt$$

$$= F_1\alpha_1T_1[Q_1(1 - \frac{\alpha_1T_1^2}{6}) - (D_1+D_2)(\frac{T_1}{2} - \frac{\alpha_1T_1^3}{12})]$$

For item 2, loss due to decay,

$$= F_2\alpha_2T_2\ [Q_2(1 - \frac{\alpha_2T_2^{\ 2}}{6}) - (D_1+D_2)(\frac{T_2}{2} - \frac{\alpha_2T_2^{\ 3}}{12})]$$

Thus Total annual cost

$$\begin{aligned}
TC \ = \ & \frac{1}{T_1+T_2}\ [A_1+C_1Q_1+A_2+C_2Q_2 + h_1Q_1\ (T_1 - \frac{\alpha_1T_1^{\ 3}}{6})\\
& + h_2Q_2\ (T_2 - \frac{\alpha_2T_2^{\ 3}}{6}) - h_1(D_1+D_2)\ (\frac{T_1^2}{2} - \frac{\alpha_1T_1^{\ 4}}{12})\\
& - h_2(D_1+D_2)\ (\frac{T_2^{\ 2}}{2} - \frac{\alpha_2T_2^{\ 4}}{12}) + V_1D_1T_2 + V_2D_2T_1\\
& + F_1\alpha_1Q_1T_1\ (1 - \frac{\alpha_1T_1^{\ 2}}{6}) + F_2\alpha_2Q_2T_2\ (1 - \frac{\alpha_2T_2^{\ 2}}{6})\\
& - F_1\alpha_1T_1\ (D_1+D_2)\ (\frac{T_1}{2} - \frac{\alpha_1T_1^{\ 3}}{12})\\
& - F_2\alpha_2T_2\ (D_1+D_2)\ (\frac{T_2}{2} - \frac{\alpha_2T_2^{\ 3}}{12})]
\end{aligned} \tag{2.18}$$

Setting the partial derivatives of TC with respect to Q_1 and Q_2 equal to zero yields,

$$\frac{\partial TC}{\partial Q_1} = (v\frac{\partial u}{\partial Q_1} - u\frac{\partial v}{\partial Q_1})/v^2 = 0 \tag{2.19}$$

and,

$$\frac{\partial TC}{\partial Q_2} = (v\frac{\partial u}{\partial Q_2} - u\frac{\partial v}{\partial Q_2})/v^2 = 0 \tag{2.20}$$

where,

$$v = T_1+T_2$$

$$\begin{aligned}
u \ = \ & A_1+A_2 + C_1Q_1 + C_2Q_2 + h_1Q_1\ (T_1 - \frac{\alpha_1T_1^{\ 3}}{6})\\
& + h_2Q_2\ (T_2 - \frac{\alpha_2T_2^{\ 3}}{6}) - h_1(D_1+D_2)\ (\frac{T_1^{\ 2}}{2} - \frac{\alpha_1T_1^{\ 4}}{12})\\
& - h_2\ (D_1+D_2)\ (\frac{T_2^{\ 2}}{2} - \frac{\alpha_2T_2^{\ 4}}{12}) + V_1D_1T_2
\end{aligned}$$

$$
\begin{aligned}
&+ V_2D_2T_1 + F_1\alpha_1Q_1T_1\left(1 - \frac{\alpha_2T_1^{\,2}}{6}\right) \\
&+ F_2\alpha_2Q_2T_2\left(1 - \frac{\alpha_2T_2^{\,2}}{6}\right) \\
&- F_1\alpha_1T_1\,(D_1+D_2)\left(\frac{T_1}{2} - \frac{\alpha_1T_1^{\,3}}{12}\right) \\
&- F_2\alpha_2T_2\,(D_1+D_2)\left(\frac{T_2}{2} - \frac{\alpha_2T_2^{\,3}}{12}\right)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial u}{\partial Q_1} = {} & C_1 + h_1\left(T_1 - \frac{\alpha_1T_1^{\,3}}{6}\right) + h_1Q_1\left(1 - \frac{3\alpha_1T_1^{\,2}}{6}\right)\frac{T_1}{Q_1} \\
&- h_1\,(D_1+D_2)\left(T_1 - \frac{4\alpha_1T_1^{\,3}}{12}\right)\frac{T_1}{Q_1} \\
&+ V_2D_2\,\frac{T_1}{Q_1} + F_1\alpha_1\left(T_1 - \frac{\alpha_1T_1^{\,3}}{6}\right) \\
&+ F_1\alpha_1Q_1\left(1 - \frac{3\alpha_1T_1^{\,2}}{6}\right)\frac{T_1}{Q_1} \\
&- F_1\alpha_1\,(D_1+D_2)\left[T_1 - \frac{4\alpha_1T_1^{\,3}}{12}\right]\frac{T_1}{Q_1}
\end{aligned}
\tag{2.21}
$$

$$
\begin{aligned}
\frac{\partial u}{\partial Q_2} = {} & C_2 + h_2\left(T_2 - \frac{\alpha_2T_2^{\,3}}{6}\right) + h_2Q_2\left(1 - \frac{3\alpha_2T_2^{\,2}}{6}\right)\frac{T_2}{Q_2} \\
&- h_2\,(D_1+D_2)\left(T_2 - \frac{4\alpha_2T_2^{\,3}}{12}\right)\frac{T_2}{Q_2} + V_1D_1\,\frac{T_2}{Q_2} \\
&+ F_2\alpha_2T_2\left(1 - \frac{\alpha_2T_2^{\,2}}{6}\right) + F_2\alpha_2Q_2\left(1 - \frac{3\alpha_2T_2^{\,2}}{6}\right)\frac{T_2}{Q_2} \\
&- F_2\alpha_2\,(D_1+D_2)\left(T_2 - \frac{4\alpha_2T_2^{\,3}}{12}\right)\frac{T_2}{Q_2}
\end{aligned}
\tag{2.22}
$$

$$\frac{\partial v}{\partial Q_1} = \frac{\partial(T_1+T_2)}{\partial Q_1} = \frac{dT_1}{dQ_1} \qquad (2.23)$$

$$\frac{\partial v}{\partial Q_2} = \frac{\partial(T_1+T_2)}{\partial Q_2} = \frac{dT_2}{dQ_2} \qquad (2.24)$$

where,

$$\frac{dT_1}{dQ_1} = -\frac{1}{2(D_1+D_2)} + y_1 \frac{(A_1+B_1)}{Q_1} - z_1 \frac{(A_1-B_1)}{Q_1} \qquad (2.25)$$

$$\begin{aligned}\frac{d(A_1+B_1)}{dQ_1} = {} & 2\Big[\, \tfrac{1}{6} (A_{11}^2 + B_{11}^2)^{-5/6} \\ & (2A_{11}(3K_1Q_1{}^2+K_2) - (4K_4Q_1{}^3+2K_5Q_1)) \\ & \mathrm{Cos}\ (\tfrac{C}{3}) + (A_{11}^2+B_{11}^2)^{1/6}\ (-\mathrm{Sin}\ (\tfrac{C}{3}))\ \tfrac{1}{3} \\ & (1+ (\tfrac{B_{11}}{A_{11}})^2)^{-1}\ ((A_{11} \tfrac{B_{11}}{Q_1} - B_{11} \tfrac{A_{11}}{Q_1})/A_{11}^2)\Big]\end{aligned} \qquad (2.26)$$

and,

$$\begin{aligned}\frac{d(A_1-B_1)}{dQ_1} = {} & 2\Big[\, \tfrac{1}{6} (A_{11}^2+B_{11}^2)^{-5/6} \\ & (2A_{11}(3K_1Q_1{}^2+K_2) - (4K_4Q_1{}^3+2K_5Q_1)) \\ & \mathrm{Sin}\ (\tfrac{C}{3}) + \tfrac{1}{3} (A_{11}^2+B_{11}^2)^{1/6}\ \mathrm{Cos}\ (\tfrac{C}{3}) \\ & ((1+(\tfrac{B_{11}}{A_{11}})^2)^{-1})\ (A_{11} \tfrac{B_{11}}{Q_1} - B_{11} \tfrac{A_{11}}{Q_1})/A_{11}^2\Big]\end{aligned} \qquad (2.27)$$

where,

$$\frac{dA_{11}}{dQ_1} = 3K_1Q_1^{\,2} + K_2 \tag{2.28}$$

$$\frac{dB_{11}}{dQ_1} = -(4K_4Q_1^{\,3} + 2K_5 - Q_2)/(2B_{21}) \tag{2.29}$$

$$C = \tan^{-1}\left(\frac{B_{11}}{A_{11}}\right) \tag{2.30}$$

Similarly,

$$\frac{dT_2}{dQ_2} = -\frac{1}{2(D_1+D_2)} + y_2\,\frac{(A_{21}+B_{21})}{Q_2} - z_2\,\frac{(A_{21}-B_{21})}{Q_2} \tag{2.31}$$

where,

$$\begin{aligned}\frac{d(A_{21}+B_{21})}{dQ_2} = {} & 2\,[\,\tfrac{1}{6}\,(A_{21}^2 + B_{21}^2)^{-5/6} \\ & (2A_{21}\,(3R_1Q_2^{\,2} + R_2) - (4R_4Q_2^{\,3} + 2R_5Q_2)) \\ & \mathrm{Cos}\,(\tfrac{C_1}{3}) + (A_{21}^2 + B_{21}^2)^{1/6}\,(-\mathrm{Sin}\,(\tfrac{C_1}{3}))\,\tfrac{1}{3} \\ & ((1+(\tfrac{B_{21}}{A_{21}})^2)^{-1})\,((A_{21}\,\tfrac{B_{21}}{Q_2} - B_{21}\,\tfrac{A_{21}}{Q_2})/A_{21}^2)]\end{aligned} \tag{2.32}$$

where,

$$\frac{dA_{21}}{dQ_2} = 3R_1\,Q_2^{\,2} + R_2 \tag{2.33}$$

$$\frac{dB_{21}}{dQ_2} = -(4R_4\,Q_2^{\,3} + 2R_5Q_2)/(2B_{21}) \tag{2.34}$$

$$C_1 = \tan^{-1}\left(\frac{B_{21}}{A_{21}}\right) \tag{2.35}$$

Using eqns. (2.21) - (2.35), and substituting the terms in eqns. (2.19) and (2.20) we get,

$$\frac{\partial TC}{\partial Q_1} = f_1(Q_1, Q_2)$$

$$= 0 \qquad (2.36)$$

$$\frac{\partial TC}{\partial Q_2} = f_2(Q_1, Q_2)$$

$$= 0 \qquad (2.37)$$

Solving the non-linear equations (2.36) and (2.37) by simultaneous iterative method for various values of Q_1 and Q_2, we get various combinations of Q_1, Q_2, T_1 and T_2, which will give local optimum solutions. The values of Q_1, Q_2, T_1 and T_2 which give global optimum are found.

One problem is solved and the results are tabulated. Exhaustive enumeration method is followed, because:

(i) Two local minimum points might be giving same TC values but one might give at low values of Q_1, Q_2, T_1 and T_2 comparing to the other. In this, manufacturers will give preference to the point which comes at low values of decision variables.

(ii) Many a time optimal point will not be at $\frac{\partial TC}{\partial Q_1} = 0$ and/or $\frac{\partial TC}{\partial Q_2} = 0$. So only scanning the whole function will be the best approach.

(iii) Problem in using non-linear programming techniques is that finding initial points to start iteration is found to be difficult.

Table 2.1: Total cost vs. Q_1 and Q_2.

Q_1	Q_2	Total Cost	T_1	T_2
130.00	10050.00	5728.48	26.67	60.07
150.00	10050.00	2891.82	26.563	60.07
170.00	10050.00	3938.75	26.46	60.07
190.00	10050.00	4989.10	26.35	60.07
210.00	10050.00	6042.82	26.24	60.07
230.00	10050.00	7099.83	26.13	60.07

Total cost is minimum at $Q_1 = 150$, $Q_2 = 10050$,
$T_1 = 26.7$, $T_2 = 60.07$.

CHAPTER III

SIMULATION MODEL FOR ONE WAY SUBSTITUTION

3.1 NATURE OF THE PROBLEM:

Let us consider a manufacturing situation where end-item is in a fluctuating demand. Mathematical models show that there could be an error as high as 30 percent in the forecast of the demand for the item. We assume that the product can be substituted by some other available item with similar characteristics.

Assume that instead of buying it at the moment the demand exceeds inventory level, the industry plans to keep an inventory of the product which is going to substitute the original. To tackle the problem of non-availability of space and to achieve some economy by making use of the difference in cost of First level items (end-items to customers) and second level items (items which are one stage prior to the end-items), Assume that the plan is to buy the end-item substitute and/or second level items. So the company has to rework on second level items before assembling it to make the end-item substitute, if second level items are also purchased.

The selection of buying end-item substitute and/or second level items depends on the cost of end-items, end-item substitute, second level items, holding cost of first and second level items, emergency set-up costs, and cost of reworking and assembling end-item substitute from second level items.

Our objective is to meet a particular service level by substitution at an economical inventory carrying cost.

3.2 LITERATURE REVIEW:

Many attempts have been made by researchers to solve the problem of unsatisfied demand by substituting it with an item of similar characteristics. In a paper by Carlson and Yano [1], the formulation of an algorithm for fixing buffer levels taking into account the uncertainty of demand in MRP systems is being quoted. Eventhough there are many mathematical models for substitution of demand developed by Nigam [5], easy-to-use, simple and cost-effective methods are lacking in this area.

In practical situations, it is very difficult to represent a system with many constraints putforth by market conditions and organisation policies mathematically, because of the analytical complexity involved in modelling. With this in view, in this chapter a simulation model is developed to find the optimal inventory policy of an inventory system taking into account the factors like emergency set-up cost, reworking cost and inventory holding cost of end-item substitute and second level items at a specified service level. Numerical examples are given and results analysed.

3.3 SYSTEM DESCRIPTION:

As there are three different situations assumed, we consider here three different ways of working of the system. They are:

(1) Situation where reworking to make end-item and emergency ordering of end-items proves to be economical. In this situation

the required quantity of end-items and second level items are procured. The customer's demand for item considered is fulfilled with the same, and side by side, the end-item substitute will be in the making from second level items. If the demand for end-items exceeds, it will be fulfilled by the substitute which is made available by reworking and assembling. But if the demand goes beyond the procured quantity, we go for emergency ordering of end-item substitute. If enough time is left, emergency set-up is carried out till fill-rate crosses the required service level.

In case of actual demand lesser than procured quantity, the excess is carried over to next period and that quantity of excess items will be reduced in the quantity to be ordered for third period.

This procedure is followed subsequently.

(2) Situation where stocking of end-item substitute and its emergency ordering proves economical.

In this situation only end-items and end-item substitutes are procured in inventory. Initially the customer demand is fulfilled by the end-item and then by end-item substitute. If the demand goes excessively, emergency set-ups are made.

In case of demand lesser than inventory, excessive end-items and end-item substitutes are carried over to next period and the ordering quantity for third period is accordingly adjusted. This follows for subsequent cycles too.

(3) Situation where stocking of end-item and end-item substitute proves to be economical:

Demand is fulfilled by end-item, end-item substitute

Inventory carrying and/or emergency set-ups are as in cases 1 and 2.

3.4 ASSUMPTIONS:

Following assumptions are made to characterise the model.

1. One end-item is made of two second level items each.
2. Demand forecast for end-item in all periods of the planning horizon is available.
3. Demand for end-item are in critical situation.
4. Individual demands for second level items are not considered.
5. The forecast of demand is equal to the mean demand and forecast errors are assumed to be distributed normally with mean as zero.
6. The standard deviation of the forecast errors for an item in a period is assumed to be a constant fraction of the mean demand of that item in that period.
7. The holding costs for a period are charged on the inventory carried over from that period to next.
8. No other uncertainty exists in the system. Lead times and quantity supplied are assumed to be deterministic.
9. Emergency set-ups are allowed only to procure end-level items.
10. Lead time for emergency set-ups are finite and lesser than the lead time in case of regular set-ups.
11. Emergency set-up cost is higher than set-up costs per order

which are encountered in regular procurements.

3.5 SERVICE LEVEL AND STANDARD DEVIATION:

There are two principal approaches to the selection of safety stock, namely specification of a desired service level and explicit costing of shortages. Two popular measures of the former are:

(i) Specified fraction of demand to be satisfied routinely from stock which is also called the fill-rate.

(ii) Specified probability of no-stockout per replenishment cycle.

We will be using the former one because of its practicality and easy-to-use nature.

As forecast errors are very difficult to estimate even under stationary or slowly changing situations, we assume that the standard deviation of forecast errors is a constant fraction of the demand forecast which is the mean of the distribution. This has been reported to serve well in many situations by Silver [7]. To make the problem tractable a relatively simple method to account for the changing nature of demand and hence the assumption.

3.6 NOTATIONS:

C - Fraction of demand giving the standard deviation.

T - Number of periods in the planning horizon.

K(i) - Safety factor for item i, to achieve desired service level where i = 1 for end-item substitute,
i = 2 or 3 for second level items.

D_{ij} - Mean of the demand for item i in period j.

σ_{ij} - Standard deviation of demand for item i in period j.

SS_{ij} - Safety stock required for item i in period j.

S_{ij} - Order upto level for item i in period j.

a_e - Major set-up cost for end-item.

b_e - Minor set-up cost for end-item.

b_i - Minor set-up cost for end-item substitute (i=1), second level items (i = 2,3).

h_e - Inventory holding cost for one unit of end-item.

h_i - Inventory holding cost for one unit of end-item substitute (i=1), second level items (i=2,3).

Q_{ij} - Quantity of item i to be ordered in period j.

$ONHAND_{ij}$ - Quantity of end-item i on hand in period j.

REWORK - Cost of reworking involved in making end-item from second level items in Rs./unit of end-item substitute.

EMRGEN - Emergency set-up cost in ordering end-item substitute in case of emergency in Rs./N items.

An emergency order for N items or below, emergency set-up cost will be one EMRGEN. For an emergency order for X items where $N < X \leq 2N$, the emergency set-up cost being two EMRGEN and so on.

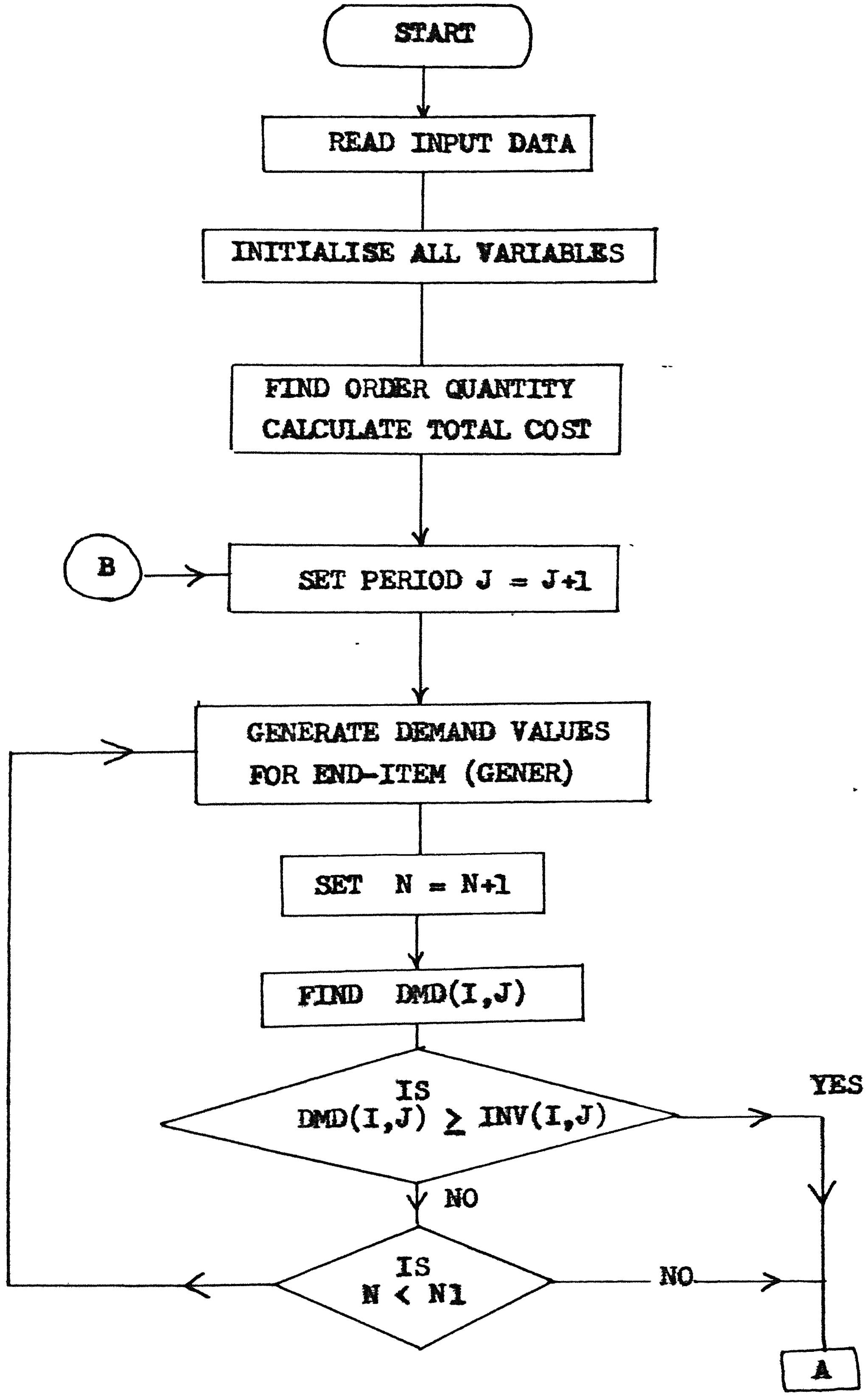


FIG. 3.1: FLOW CHART FOR SIMULATION MODEL.

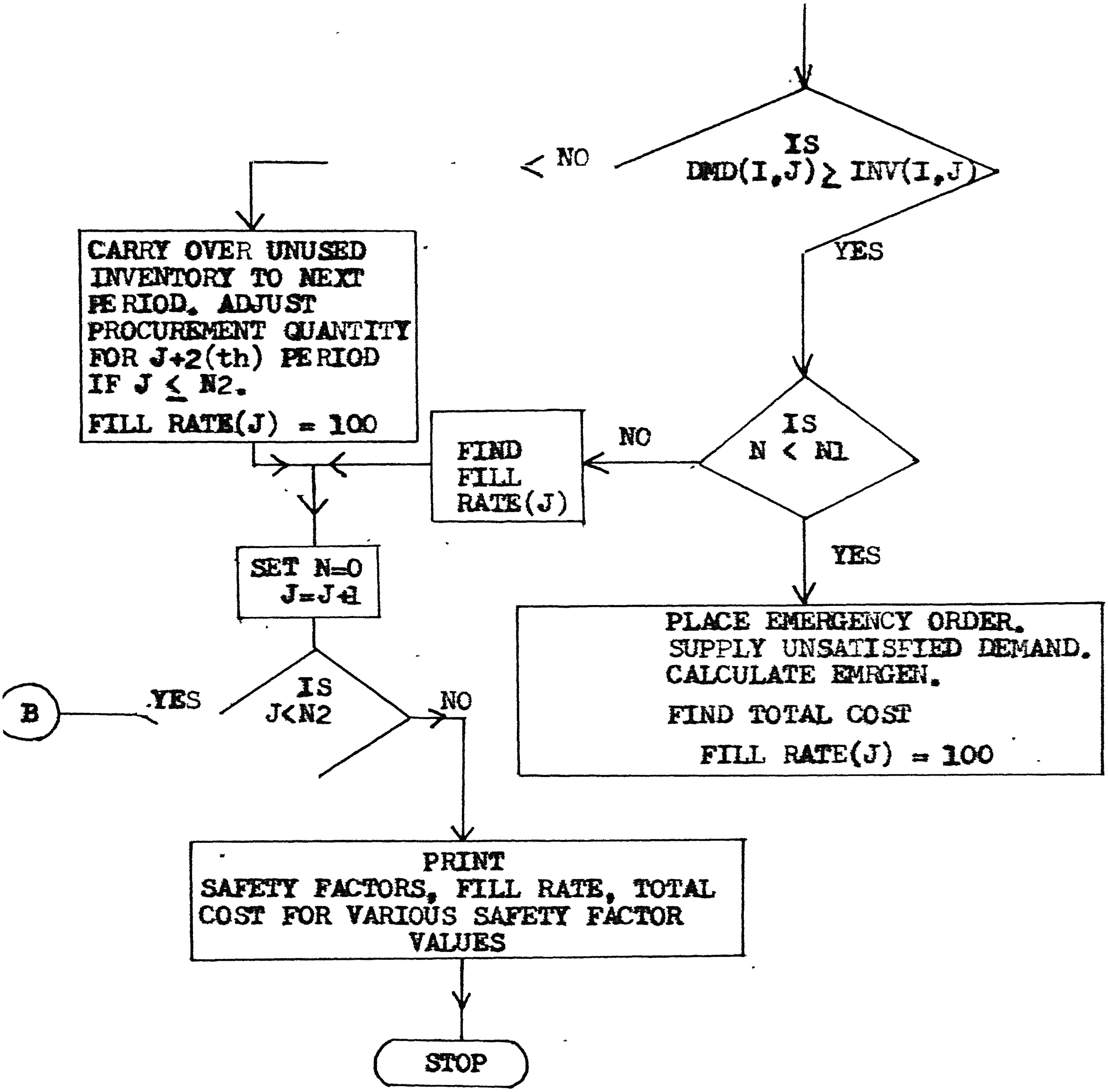

IS
DMD(I,J) ≥ INV(I,J)
NO
YES
CARRY OVER UNUSED INVENTORY TO NEXT PERIOD. ADJUST PROCUREMENT QUANTITY FOR J+2(th) PERIOD IF J ≤ N2.
FILL RATE(J) = 100
IS
N < N1
NO
FIND FILL RATE(J)
YES
PLACE EMERGENCY ORDER. SUPPLY UNSATISFIED DEMAND. CALCULATE EMRGEN.
FIND TOTAL COST
FILL RATE(J) = 100
SET N=0
J=J+1
B
YES
IS
J<N2
NO
PRINT
SAFETY FACTORS, FILL RATE, TOTAL COST FOR VARIOUS SAFETY FACTOR VALUES
STOP

Fig. 3.1(a): BLOCK A OF FIG. 3.1.

3.7 HEURISTIC ALGORITHM:

Joint lot sizing algorithm,(a simple version of lot-sizing algorithm (Wagner and Whitin [8])) which was developed by Gopinathan [4] is used along with Dynamic Rolling Schedule. The functioning of Dynamic Rolling Schedule is given in system description, whereas the heuristic algorithm is given in Appendix A.

3.8 SIMULATION MODEL:

A Simulation Program is written in FORTRAN with subroutin HEURIS and GENER, to find the optimal inventory policy of how much to procure and which type of items to be procured to meet the required service level. Fig. (3 .1) shows the flow chart of simulation model.

HEURIS:

This subroutine finds quantity to be procured in terms of end-item, end-item substitute and/or second level items by applying the heuristics mentioned earlier in this chapter. This is done before applying Dynamic Rolling Schedule.

GENER:

This subroutine generates Random Numbers (which are distributed normally) to find actual demand for the given mean demand and standard deviations. A system supplied Random Number Generator G05DDF from NAG is used by this subroutine.

Objective Functions:

Objective is to find the optimal ordering policy, given the reworking costs, emergency set-up cost, major and minor set-up costs for a required service level.

3.9 SIMULATION EXPERIMENT:

Problems are solved using the simulation model described above with different cost parameters for an inventory system of

Planning Horizon	1 year
Number of periods	12
Emergency set-up time	1/5th of a month
Fill-rate	90 percent of demand.

The input parameters for the problems are given in Table 3.1, and 3.2 and the output results are shown in Table 3.3 and 3.4.

From the output results the following observations are made.

1) As the reworking cost is low, it is obvious that better to maintain second level item inventory instead of going for end-item or end-item substitute inventory, and emergency set-ups. The graph (Fig. 3.3) shows that the effect of K(1) and K(2) on total cost, at 90 percent fill rate.

2) For the cost structure given in problem 2, at zero level safety stock for all items, it is found that emergency set-up is economical, to maintain a fill-rate of 90 percent. Fig. 3.4 shows at zero level safety stock of second level items, and with an

increase in safety stock of end-item inventory, there is a decrease in total annual cost. The trade-off between emergency set-up cost and inventory holding cost is at K(1) = 5 percent and after that the total annual cost started rising with increase in safety stock level of end-item substitute.

Table 3.1: Input values for the Problem.

Cost terms	Rs.
Major set-up cost	50.00
Minor Set-up Cost for	
End-item	25.00
End-item Substitute	25.00
Second level item 1	15.00
Second level item 2	10.00
Holding Cost for	
End-item	1.10
End-item Substitute	1.10
Second level item 1	0.55
Second level item 2	0.40

Table 3.2: Input values for the problem.

Problem No.	Reworking Cost	Emergency Cost
1	0.6	1.4
2	1.4	1.4

Table 3.3: Total cost vs. K(1) and K(2) for Problem 1, (in Rs.).

K(1) \ K(2)	Total Cost (Rs.)		
	0	5	10
0	1882.28	2748.63	3388.55
5	1385.18	2791.53	3431.45
10	1423.13	2829.48	3469.40
15	1527.68	2874.03	3513.45
20	1626.73	2913.08	3553.00
25	1729.63	2955.98	-

Table 3.4: Total cost vs. K(1) and K(2) for Problem 2 (in Rs.).

K(2) \ K(1)	0	5	10
	Total cost (Rs.)		
0	1665.75	1702.23	1733.95
5	1577.10	1613.67	1645.30
10	1607.03	1643.50	1675.22
15	1637.42	1673.95	1705.63
20	1665.93	1702.33	1734.13
25	1697.27	1733.67	-
30	1729.20	1763.50	-

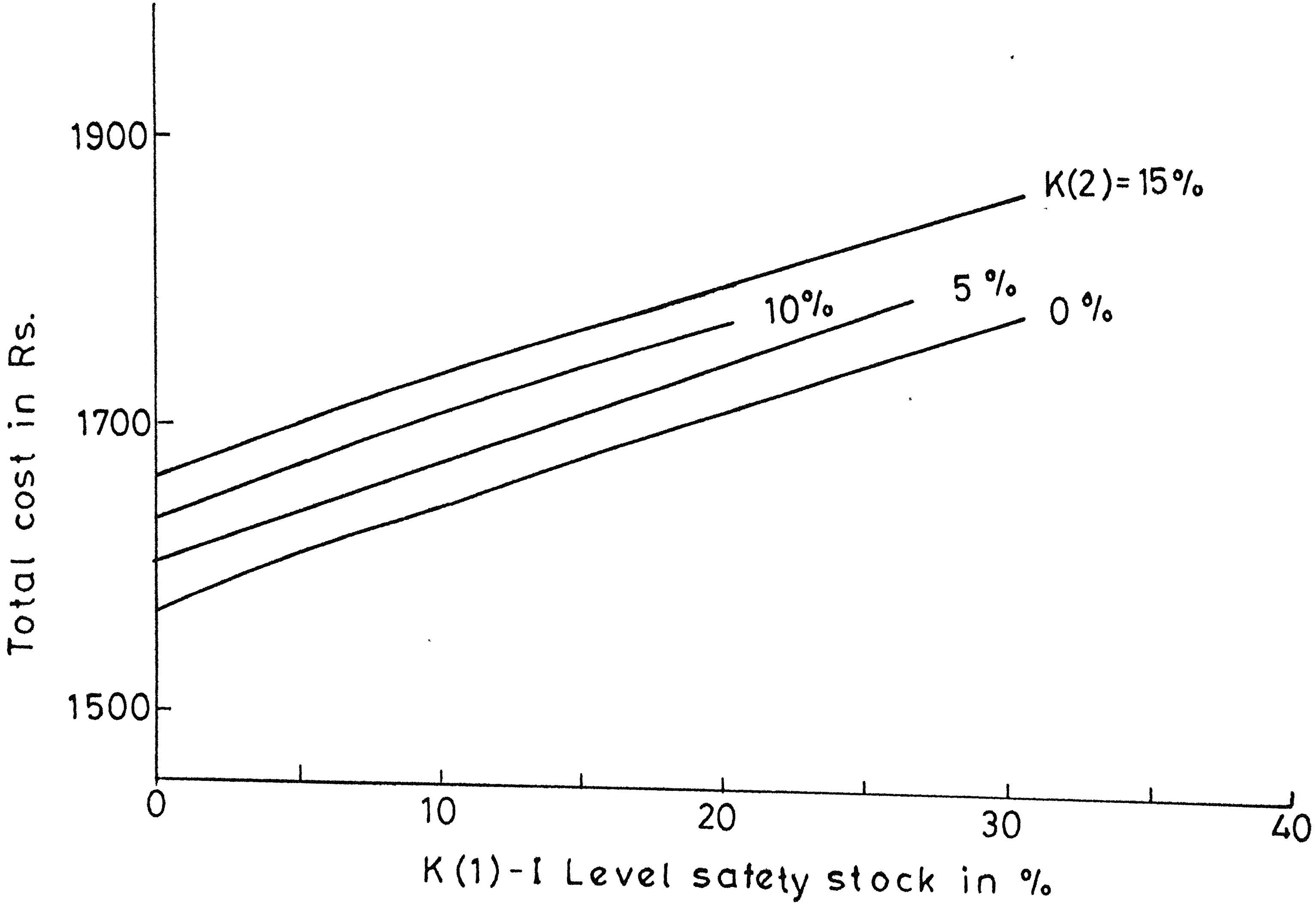


Fig. 3·2 Effect of K(1) and K(2) on total cost

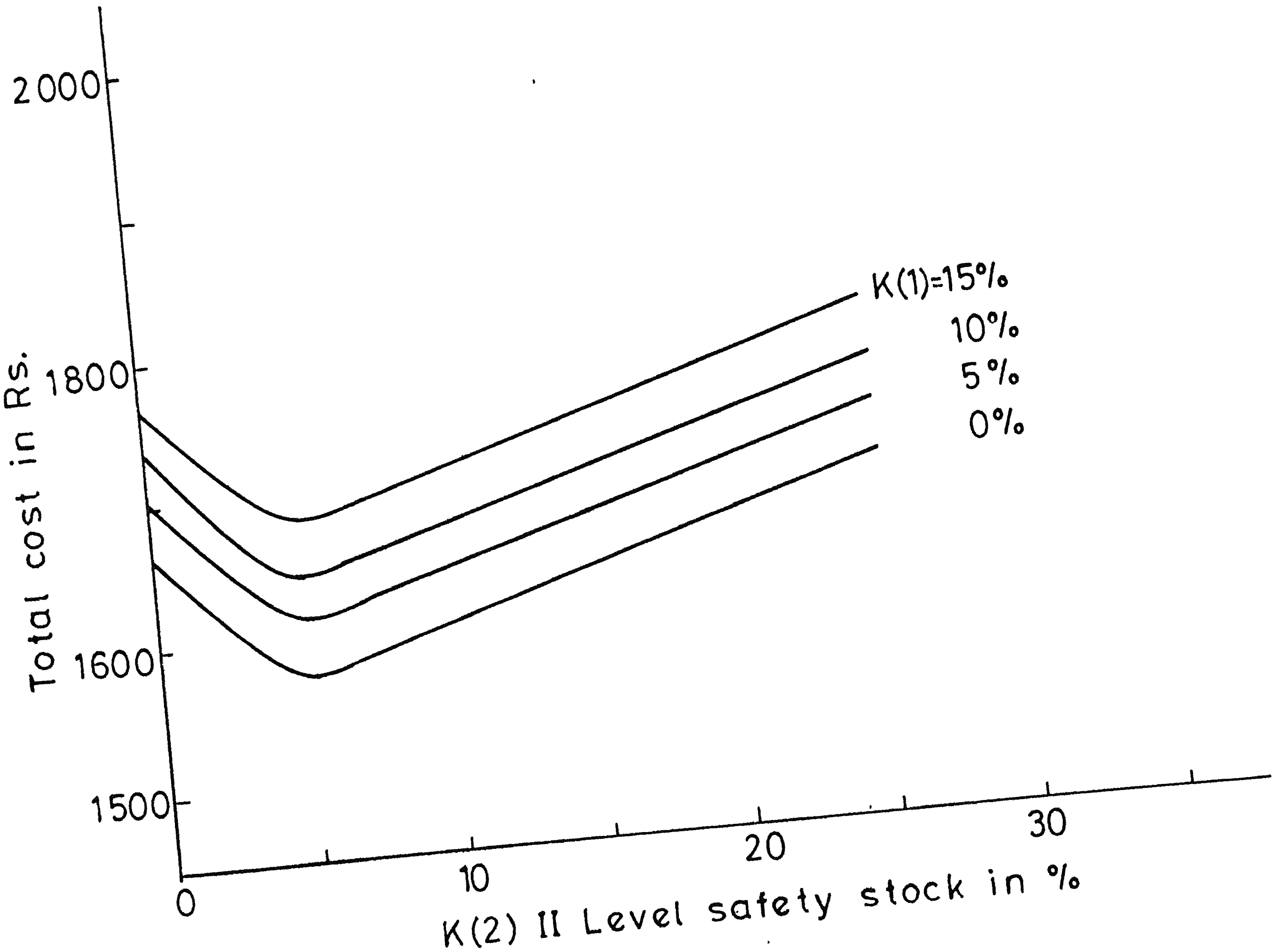


Fig.3·3 Effect of K(1) and K(2) on total cost

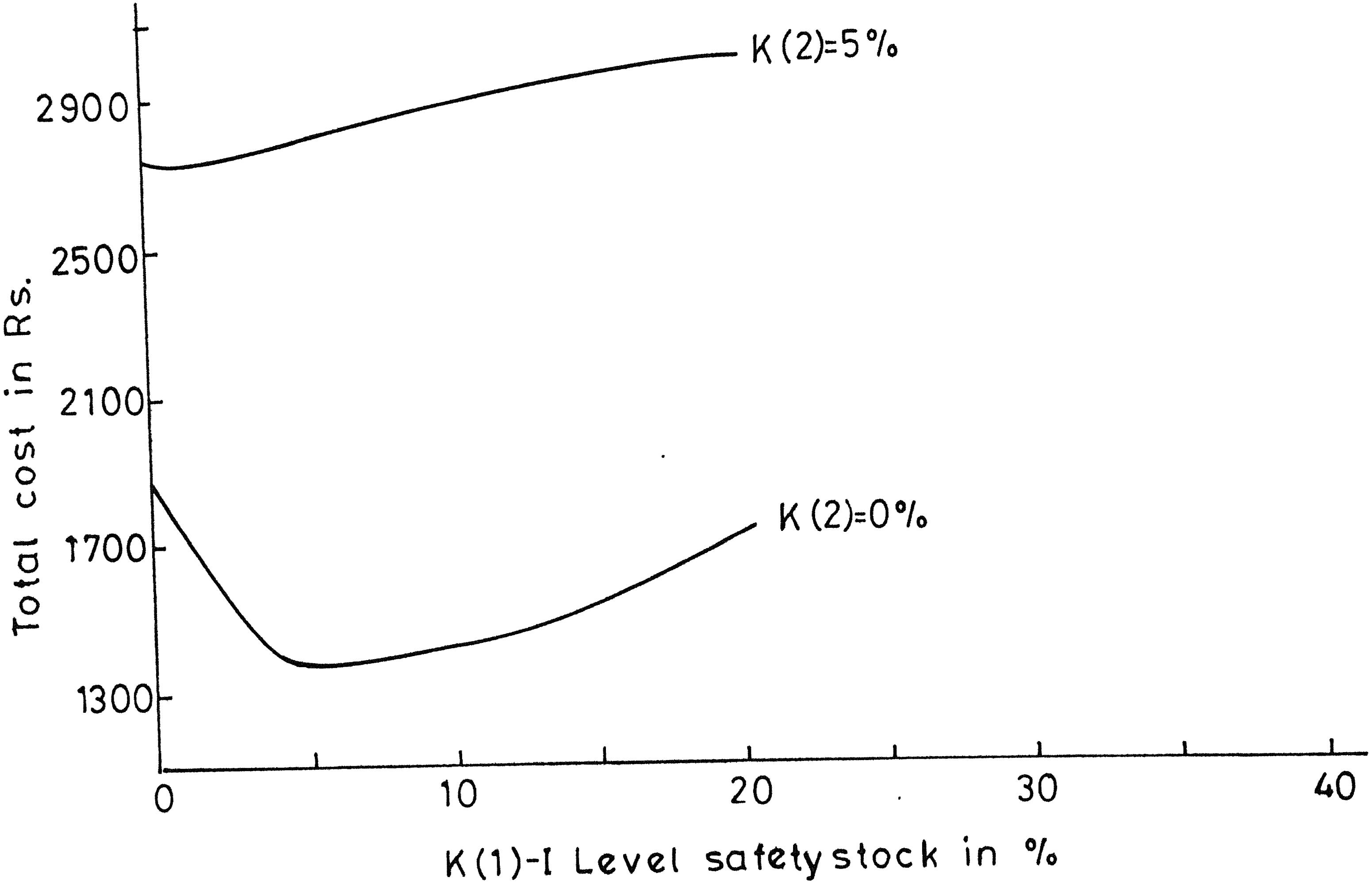


Fig. 3·4 Effect of K(1) and K(2) on total cost

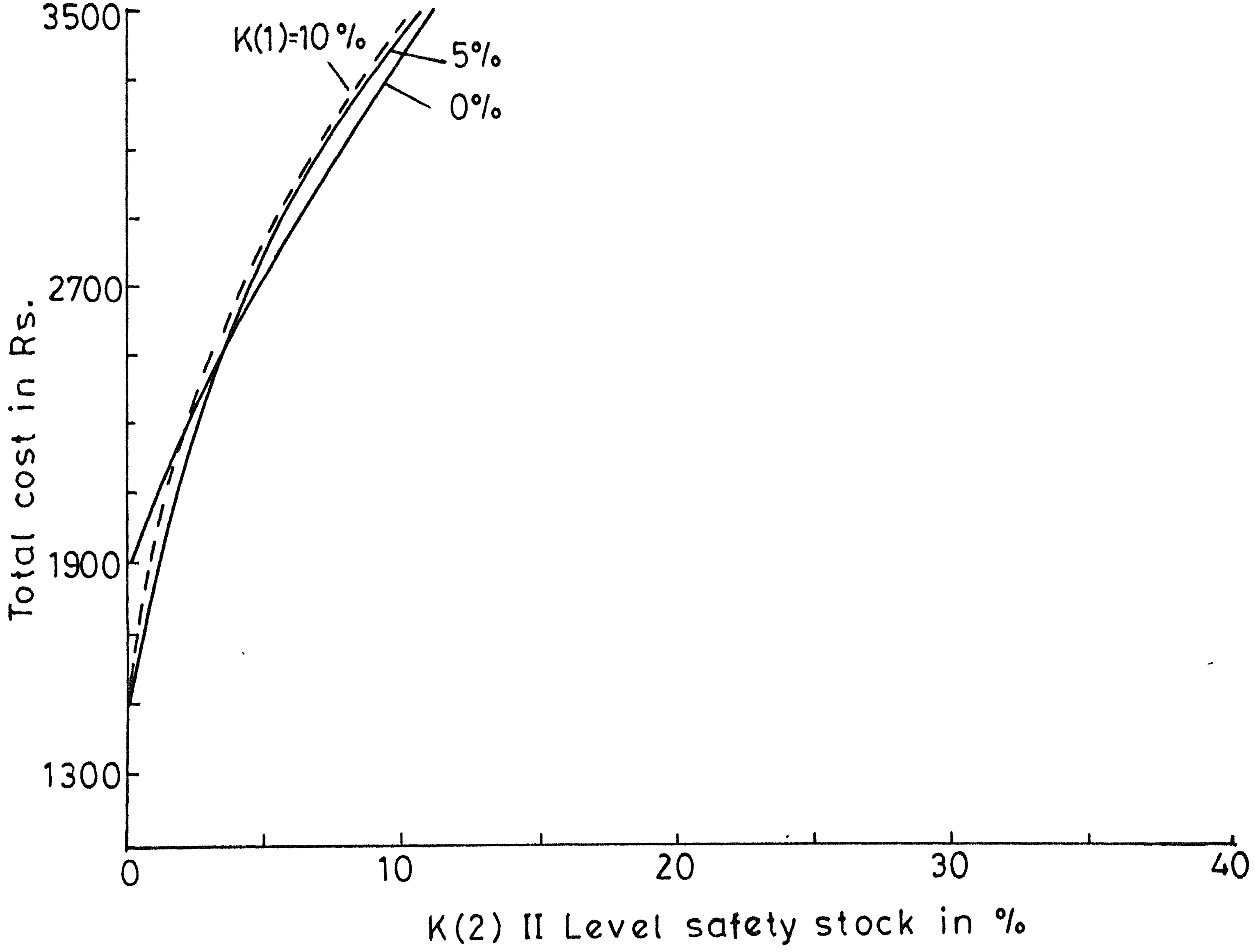


Fig.3·5 Effect of K(1) and K(2) on total cost

CHAPTER IV

SIMULATION FOR N-ITEM SUBSTITUTION

4.1 PROBLEM DEFINITION:

We consider an inventory system of N-items. We further assume that among these items some items are of interchangeable in nature. It means, in case of stock-out of one item, its demand can be fulfilled with some other item(s) of similar characteristics. If no such item is available, its demand is backlogged and fulfilled in the next period.

In this chapter, a simulation model is developed for widely used periodic review model, generally called (Q,r,T) model. As this model precludes an analytical approach, with substitution, simulation technique seems to be a suitable method in evaluating the decision variables of the model.

The objective is to find the values of Q, r, R and T of the (Q,r,T) model at minimum total cost with N-item substitution for the given set of cost parameters.

4.2 (Q,r,T) MODEL:

The decision variables of the model are:

Q - Quantity to be ordered whenever an order is placed.

r - Reorder level - the quantity which determines the need for ordering or not ordering.

T - Cycle time - a stipulated period after which the status of inventory is reviewed.

After a time period T, inventory is reviewed and if the inventory position is below reorder level, an order for quantity Q is placed.

The (Q,r,T) model for N-item with two-way substitution can be explained as follows. Whenever an item I_D is demanded, it is satisfied if it is in stock, otherwise it is satisfied by any item I_S, the most economical of the items that can be used for substituting where $S = 1,2,\ldots,n$ and $D \neq S$, from set of n items. If the item I_D's demand cannot be satisfied fully by itself or by any other item, then the remaining quantity is backlogged.

4.3 ASSUMPTIONS:

Following assumptions are made in characterising the model:

1) Only certain items can be substituted for other item of similar characteristics.
2) The unsatisfied demand for an item is fulfilled by substitution if possible. The unfulfilled demand is backlogged.
3) Leadtime for procurement is static and known.
4) Parameters of the distribution functions are known.
5) Demands are stochastic and follow geometric distributions.
6) Inter-arrival time for item - demand is exponential.
7) Planning horizon is known

Other aspects considered are discussed in the following sections.

Can-Order Level:

When the inventory is reviewed, if the inventory position is near to Re-order level but not below it, according to (Q,r,T) conventional models, an order is not placed for it. It is obvious that in a N-item inventory system, if order is not placed for it, it may increase the substitution cost as well as backlog cost. Hence can-order level is also considered. At the time of review, if the inventory position is below can-order level and/or above reorder level, an order is placed.

Priority Order:

Three main events in the model are:

1) Customer Arrival
2) Arrival of Procurement
3) Review

When two or more events occur simultaneously, the events are selected according to the following priority rules.

1) Arrival of Procurement
2) Review
3) Customer Arrival.

Split-up of Cost:

For all items, the split-up of cost are given as,

1) Ordering cost.
2) Holding cost.
3) Backlog cost.
4) Substitution cost if applicable.

4.4 NOTATIONS:

The notations used in developing the simulation model are:

FQTY(I) - Fixed ordering quantity for item I.

TIM - Time at any point.

T - Cycle time.

REOD(I) - Reorder level for item I.

CNOR(I) - Can-order-level for item I.

HOLC(I) - Holding cost of item I.

UNTC(I) - Unit cost of item I.

BCLG(I) - Backlog cost of item I.

SUB(I,J) - Substitution cost if item I is substituted by some other item.

LEDTIM(I) - Lead time for item I.

TC(I) - Total cost involved in maintaining inventory of item I.

INV(IJ) - Inventory level of item I, in period J.

DMD(I,J) - Demand for item I in period J.

TLEFT - nT-TIM. Time left in that particular cycle n.

LREOD - Status of Reorder level.

If LREOD = 0, Order is yet to be placed for next period

LREOD > 0, Order is already placed.

PROBL - Parameter P of demand distribution.

ARVR - Parameter of inter-arrival distribution.

4.5 METHODOLOGY:

The flow chart of the simulation program is shown in Fig. 4.1, 4.1(a) and (b).

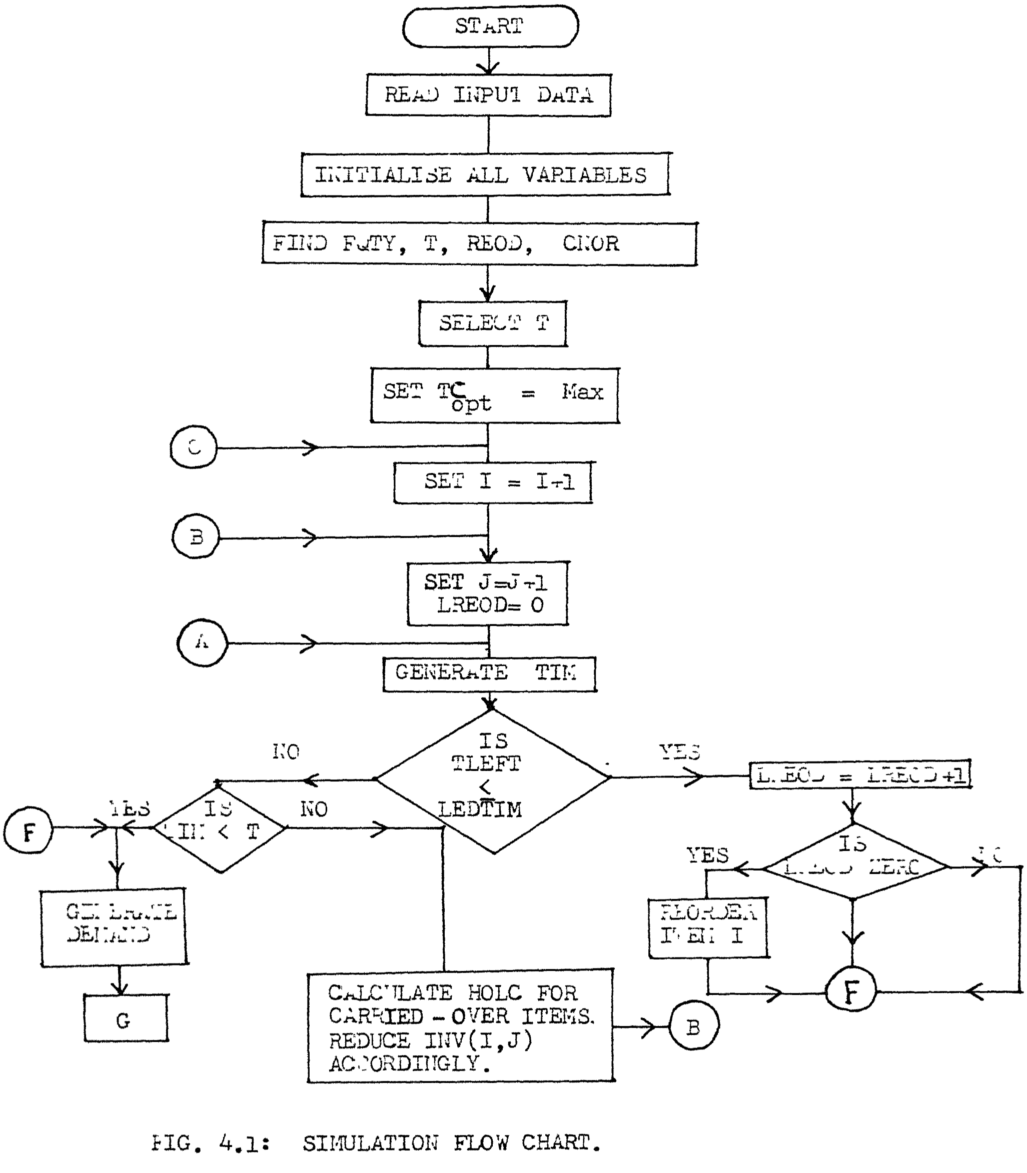


FIG. 4.1: SIMULATION FLOW CHART.

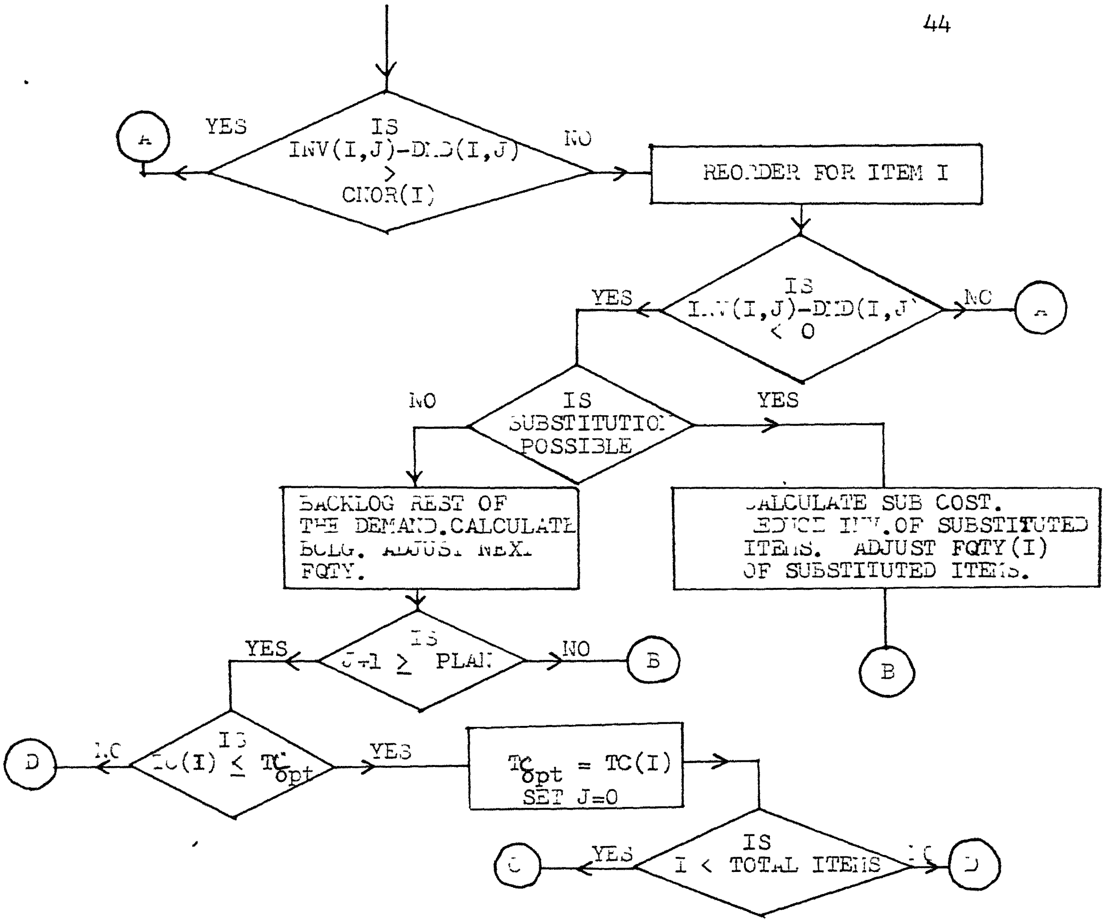


FIG. 4.1(a): BLOCK G OF FIG. 4.1.

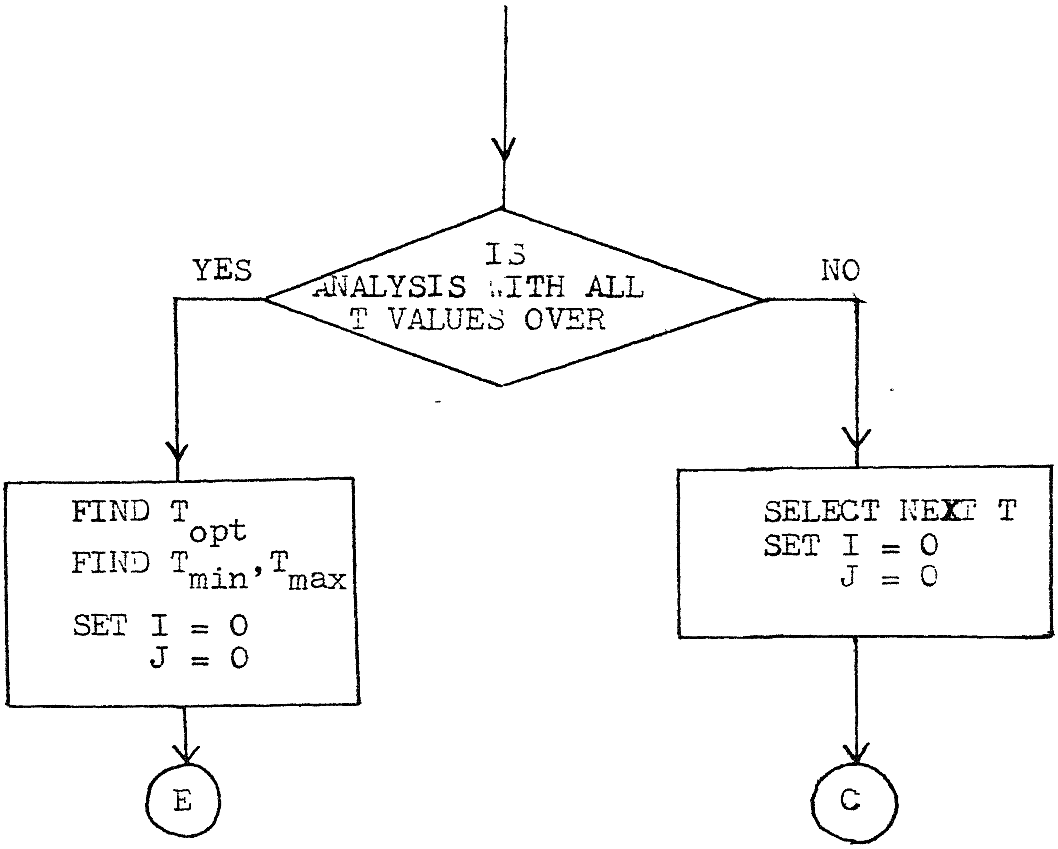


FIG. 4.1(b): BLOCK D OF FIG. 4.1(a).

To start with, the EOQ for each item is found assuming that all are ordered separately and the various time periods are calculated. This EOQ is called fixed ordering quantity, and denoted by Q^S. 10 percent of the Q^S are fixed as r^S (reorder level of item S) and 30 percent of Q^S are fixed as R^S (can-order level of item S).

Among the set of cycle times calculated for various items, minimum cycle time is chosen, and simulation is done for item I for the calculated Q,r,R values as mentioned earlier.

Simulation is actually generating time values and correspondingly demand values for item I. Time (TIM) is checked to determine the occurrance of the events in the model.

Reordering of item 1 is done
if the inventory level is below can-order level and/or
if the time left in the cycle is only just enough to
procure item 1 for next cycle in time reordered now.

If the inventory level comes below zero in a cycle due to fluctuations in demand, the item will be substituted with some other item(s). The selection of item is based on availability and economy in the cost of substitution. The unfulfilled demand is backlogged and backlogging cost is also taken into account. This is repeated in subsequent cycles for item 1. Total cost of maintaining inventory is calculated.

This procedure is repeated for all items with same cycle time. Total cost of inventorying all items is noted alongwith cycle time.

The procedure mentioned above is repeated taking different possible cycle times, and the total cost values are calculated in each case.

Block E is for the following.

The cycle time which gives minimum total cost for inventorying all items is taken. Then cycle time is perturbed on both the sides by 10 percent and the simulation is repeated to find the time period which is giving optimum total cost.

Now with the above initial parameters, the simulation is done for a planning horizon by reducing r_1 by 25 percent and then cost of inventory policy is noted each time by increasing r_1 by 1 percent, until r_1 is 1.25 times the initial r_1. The r_1 corresponding to the minimum cost is fixed as the parameter r_1. Then the simulation is done in the same way to find $r_2, r_3, \ldots, r_n$ values.

The same methodology is adopted to find $R_1, \ldots, R_n$ and $Q_1, \ldots, Q_n$.

4.6 NUMERICAL EXAMPLE:

A simulation program is written in FORTRAN to validate the model developed. A numerical example is solved and results are shown.

```
------------------------------------------------------------

    NUMBER OF ITEMS --      3
    PLANNING HORIZON -- 5.00
------------------------------------------------------------

    ITEM NO.      I FIXED COST I HOLD. COST I BACKLOG COS T I UNIT COST I
      1              35.00          4.50          2.00          10.00
      2              30.00          3.20          2.80           8.00
      3              40.00          4.30          2.80          12.00

         SUBSTITUTION MATRIX IS AS FOLLOWS

   ITEM NO I      1          2          3
------------------------------------------------------------

     1I     99.99I      0.90I      0.85I
------------------------------------------------------------

     2I      0.80I     99.99I      0.75I
------------------------------------------------------------

     3I      0.60I      0.80I     99.99I

         DISTRIBUTION DETAILS

   ITEM NO I  LEADTIME I    DEMAND   I INT-ARV-TIME I
------------------------------------------------------------

     1I        0.10I        0.92I        0.50I

------------------------------------------------------------

     2I        0.10I        0.95I        0.70I

------------------------------------------------------------

     3I        0.10I        0.99I        0.65I


          ***  END OF INPUT DETAILS  ***

------------------------------------------------------------ --

  OPTIMAL PARAMETERS OF THE POLICY WITHOUT SUBSTITUTION

------------------------------------------------------------ --

          OPTIMAL REVIEW PERIOD --  0.54  YEAR

          PLANNING HORIZON IS  5.00  YEARS

          NUMBER OF ITEMS --    3

          FOLLOWING ARE OPTIMAL QUANTITIES

          ORDERING QUANTITIES

------------------------------------------------------------ --

     ITEM NO I  QUANTITY I
```

```
--------------------------------------------------- --

--------------------------------------------------- --

        1 !      20.05  !
        2 !      25.13  !
        3 !      42.83  !
--------------------------------------------------- --

       CANORDER LEVELS

--------------------------------------------------- --

    ITEM NO !  QUANTITY !
--------------------------------------------------- --

        1 !       5.45  !
        2 !       7.24  !
        3 !      12.47  !
       REORDER LEVELS

--------------------------------------------------- --

    ITEM NO !  QUANTITY !
--------------------------------------------------- --

--------------------------------------------------- --

        1 !       2.90  !
        2 !       2.92  !
        3 !       4.68  !
--------------------------------------------------- --


        FOLLOWING ARE THE COST ESTIMATES OF THE POLICY

--------------------------------------------------- --

    TOTAL COST IS  1702.14

--------------------------------------------------- --

        COST-SPLIT-UP IS AS FOLLOWS

    ORDERING COST       660.00
    HOLDING COST        486.66
    BACKLOG COST        555.48

--------------------------------------------------- --
```

CHAPTER V

CONCLUSION AND SCOPE FOR FUTURE WORK

5.1 CONCLUSIONS:

In the present study, various deterministic as well as stochastic models have been developed for two-item/multi-item inventory system to take account of interaction of demand due to shortages and/or introduction of new items. The models take account of many realistic situations in inventory management. Some numerical examples have been taken to examine the possibility of substitution.

For deterministic demand case, the model with independent procurement of two items has been developed. The complete substitution of one by the other in case of stock-out of the former is discussed. It is found that the policy is to go for a very lengthy planning horizon. A simulation model is developed to find the optimal ordering policy for a single item case. In this model, the possibility of substituting one item by other item in one way is discussed and solution methodology is given to determine the policy of whether to go for end-item substitute or to buy second level items and reworking to make end-item, is given. It is found that the optimum ordering policy is determined not only by emergency set-up cost and reworking cost but also by service-level.

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A simulation model for N-item with two-way substitution is studied and the steps involved in finding the values of decision variables which determine the optimum inventory policy are given. It is found that, to find global optimum , several simulation runs have to be made. Substitution proved to be economical sometimes, and costlier in some other cases depending upon the cost of substitution.

5.2 SCOPE FOR FUTURE WORK:

In order to account for realistic situation, models developed in this thesis need to be extended to account for various practical situations.

Model developed in Chapter II, where two-item substitution with decay is considered, can be extended for items with different decay patterns. To be more realistic, quantity discounts can also be included.

The model for single item, one-way substitution can be extended to N-item substitution to save storage space and cost of maintaining by keeping second level items for the end-items considered. In practical cases, there will be independent demand for second level items. So models accounting for that too will be of more use in practice.

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APPENDIX A

SIMPLE VERSION OF JOINT LOT SIZING ALGORITHM

Initially lot-by-lot approach is assumed, i.e. the items needed for a particular period is purchased at the beginning of the period.

Now, starting from the last period, the trade-off between holding cost and procurement cost is made assuming that if the quantity needed for lastperiod is ordered in the penultimate period. If there is a saving in that, the items needed in last period are purchased in penultimate period. If there is no cost saving in procuring it in previous period, lot-by-lot approach is assumed for last period.

This is applied to all periods starting from end period, and economic order quantities are determined.

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